

10/26/23 Discussion:

Wanted to clarify bounds on parameterizations.

We should always think about parameterizations $r(t)$ as having bounds on t , i.e. $a \leq t \leq b$ (when one of these bounds could be $-\infty$ or ∞)

When reparameterizing r w/ a new variable s , we need to find new bounds.

In the case of arc length parameterization, we are given $r(t)$ for $a \leq t \leq b$, then:

$$s = g(t) = \int_a^t \|r'(u)\| \, du$$

and we find $g^{-1}(s)$ w/ bounds $g(a) \leq s \leq g(b)$

ex from discussion: $r(t) = \langle 3t+1, 4t-5, 2t \rangle$

Assume $0 \leq t \leq 5$. Then since $s = g(t) = \sqrt{29}t$,
our bounds on s are $g(0) = 0$ & $g(5) = \sqrt{29} \cdot 5$.

So, our new parameterization is:

$$r(g^{-1}(s)) = \left\langle \frac{3s}{\sqrt{29}} + 1, \frac{4s}{\sqrt{29}} - 5, \frac{2s}{\sqrt{29}} \right\rangle$$